B.E. / B.Tech. Instrumentation Engineering (Model Curriculum) Semester-III

IN301 - Applied Mathematics-III (Probability and Statistics)

P. Pages : 2 **GUG/S/25/13906A**

Time : Three Hours * 2 3 4 9 * Max. Marks : 80

Notes : 1. All questions are compulsory.
2. Non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$ then prove that **8**

i) $L\{tf(t)\} = -\frac{d}{ds}\bar{f}(s) = -\bar{f}^{-1}(s)$

ii) $L\{t^n \cdot f(t)\} = (-1)^n \cdot \frac{d^n}{s^n}\bar{f}(s)$

b) Find $L\left\{\frac{\sin^2 t}{t}\right\}$ and hence find $\int_0^\infty e^{-t} \cdot \frac{\sin^2 t}{t} dt$. **8**

OR

2. a) Find the L. T. of $\frac{\cos at - \cos bt}{t}$. Hence evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} \cdot dt$ **8**

b) Find $L\left\{t^2 \cos 3t\right\}$ **8**

3. a) Evaluate $L^{-1}\left\{\frac{5s-2}{s^2(s-2)(s-1)}\right\}$ **8**

b) Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)+(s^2+b^2)}\right\}$ by convolution theorem. **8**

OR

4. a) Find $L^{-1}\left\{\frac{1}{(s-2)\ (s+2)^2}\right\}$. By using convolution theorem. **8**

b) Solve by inverse L. T. $\frac{d^2x}{dt^2} + 9x = \cos 2t$, given $x(0) = 1, \times\left(\frac{\pi}{2}\right) = -1$ **8**

5. a) Find the Fourier sine transform of $e^{-|x|}$ and hence show that **8**

$\int_0^\infty \frac{\sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$, $m > 0$

b) Using Fourier integral show that **8**

$$\int_0^{\infty} \frac{\sin \pi\lambda \cdot \sin \lambda x}{1-\lambda^2}\, d\lambda = \begin{cases} \frac{\pi}{2}\sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$

OR

6. a) Solve the integral transform $\int_0^{\infty} f(x)\cdot\cos\lambda x dx = e^{-\lambda},\ \lambda > 0$ **8**

b) Find F(x) if its sine transform is $\frac{e^{-a\lambda}}{\lambda}$. Hence deduce the inverse sine transforms of $\frac{1}{\lambda}$. **8**

7. a) Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$ **8**

b) Solve $z - xp - yq = a\sqrt{x^2 + y^2 + z^2}$ **8**

OR

8. a) Solve $(D^3 - 3DD' + 2D')z = \cos(x + 2y) + e^{x+3y}$. **8**

b) Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ using separation of variable. Given $u = 3e^{-y} - 5e^{-5y}$ when x = 0. **8**

9. a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by partitioning method. **8**

b) Find the eigen values, eigen vectors and model matrix for the matrix **8**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

OR

10. a) Use Sylvester's theorem solve the matrix $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$, verify $2\sin A = (\sin 2)A$. **8**

b) Solve $\frac{dx_1}{dt} = x_1 + x_2$ and $\frac{dx_2}{dt} = x_2$ given $x_1(0) = 1,\ x_2(0) = 1$ By matrix method. **8**

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